Closing Tues: 13.2, 13.3
Closing Thur: 13.4

Exam 1 is next Thurs (April 19)
covers 12.1-12.6, 13.1-13.4
See my website for exam review.

13.2 Calculus on 3D Curves

2D Example: Consider

$$x = t, y = 2 - t^2$$

which can also be written as

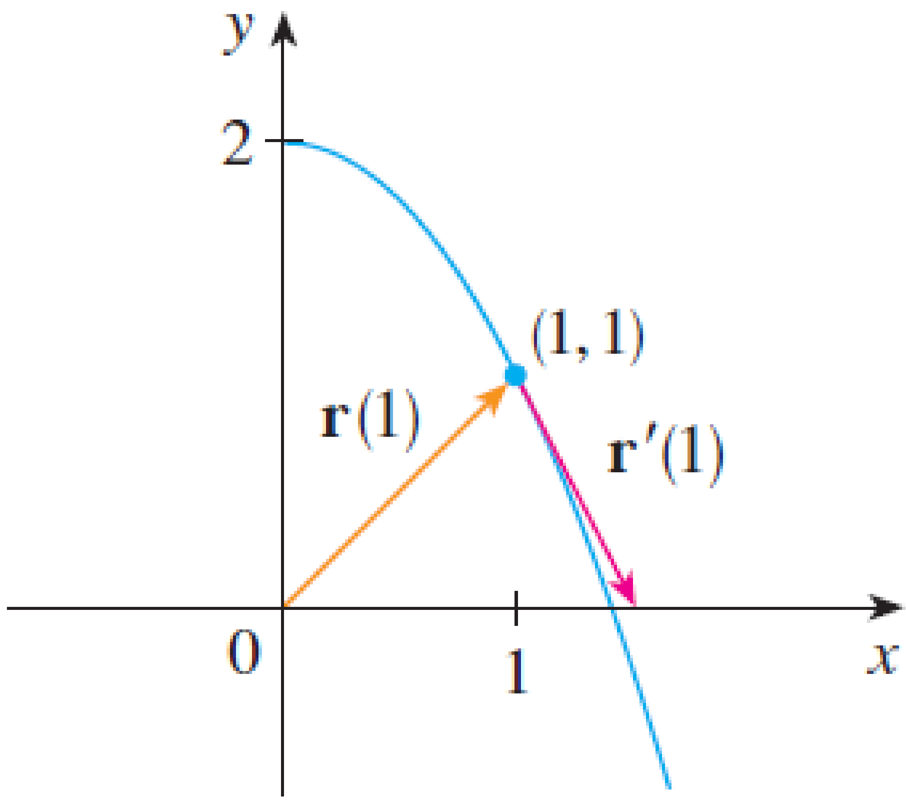
$$\boldsymbol{r}(t) = \langle t, 2 - t^2 \rangle$$

Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

When $t = 1$…
Find the location.
Find the slope of the tangent line.
Find a vector in the direction of the tangent line.

Visual of last example:

$$\boldsymbol{r}(t) = \langle t, 2 - t^2 \rangle$$



In general: Vector Calculus

For $\vec{\boldsymbol{r}}(t) = \langle x(t), y(t), z(t)\rangle$, we define

$$\vec{\boldsymbol{r}}'(t) = \lim_{h\to 0}\left\langle \frac{x(t+h)-x(t)}{h}, \frac{y(t+h)-y(t)}{h}, \frac{z(t+h)-z(t)}{h}\right\rangle$$

which is the same as

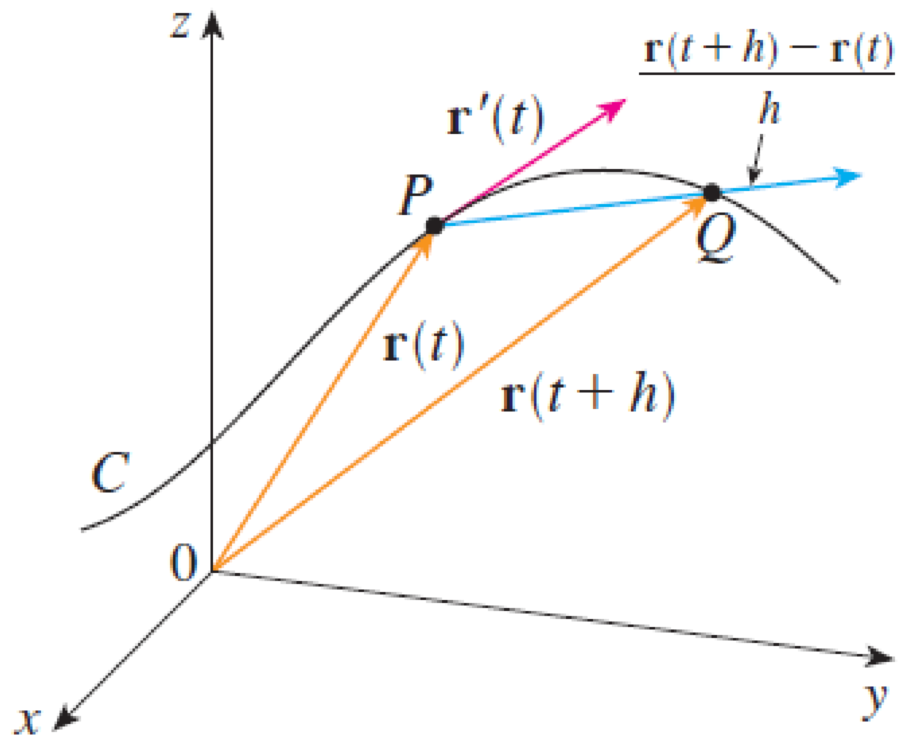
$$\vec{\boldsymbol{r}}'(t) = \langle x'(t), y'(t), z'(t)\rangle$$

And

$$\vec{\boldsymbol{r}}'(t) = \langle x'(t), y'(t), z'(t)\rangle$$

is a tangent vector to the curve.

Do calculus **component-wise**!



Example

$\vec{\boldsymbol{r}}(t) = \langle t, \cos(2t), \sin(2t) \rangle$.

1. Find $\vec{\boldsymbol{r}}'(t)$.
2. Find $\vec{\boldsymbol{r}}(0)$ and $\vec{\boldsymbol{r}}(\pi/4)$.
3. Find $\vec{\boldsymbol{r}}'(0)$ and $\vec{\boldsymbol{r}}'(\pi/4)$.
4. Find equations for the tangent line at $t = 0$.
5. Find equation for the tangent line at $t = \pi/4$

Summary of 3D calculus

$\vec{\boldsymbol{r}}'(t) = \langle x'(t), y'(t), z'(t)\rangle$ *tangent vector* (13.2)

$\int \vec{\boldsymbol{r}}(t)dt = \left\langle \int x(t)dt, \int y(t)dt, \int z(t)dt \right\rangle$ antiderivative vector (13.2/4)

13.3 Curvature, Arc Length, Normal Vector

13.4 Velocity, Speed, Acceleration (components of acceleration)

$\vec{\boldsymbol{r}}'(t) = \langle x'(t), y'(t), z'(t)\rangle$ *velocity vector* (13.4)

$|\vec{\boldsymbol{r}}'(t)| = \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2 + \left(z'(t)\right)^2}$ *speed (13.4)*

$\vec{\boldsymbol{r}}''(t) = \langle x''(t), y''(t), z''(t)\rangle$ *acceleration vector* (13.4)